

Modeling the Free Electron

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This is a computational appendix to the Chapter 3: Electron in Free Space section of R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, January 2004 Edition, posted at: <http://www.blacklightpower.com/bookdownload.shtml>. Here the free electron is modeled to scale using the physical constants and a variable relative velocity.

The free electron is a flat disk of charge whose radius (ρ_0) is proportional to Planck's constant bar, and inversely proportional to the relative velocity and electron mass, as,

$$\rho_0 = \frac{\hbar}{m_e v_z}$$

The charge distribution (σ_e) on the surface of the disk in cylindrical coordinates is a function of the radius (ρ), with the highest charge in the center and a slow taper until it drops off to zero when $\rho = \rho_0$. When the velocity goes to the speed of light ($v_z \rightarrow c$) the electron behaves as a point particle. When it goes to zero (as $v_z \rightarrow 0$), it behaves as a uniformly charged plane wave. The equation used to model the three-dimensional charge distribution is given below, and stored in "ChargeDens."

$$\sigma_e(\rho, \phi, \mathbf{z}) = \frac{e}{\frac{2}{3} \pi \rho_0^3} \sqrt{\rho_0^2 - \rho^2} \delta(\mathbf{z})$$

The physical constants are provided by Mathematica,

PlanckConstantReduced

1.05457 × 10⁻³⁴ Joule Second

ElectronMass

9.10938 × 10⁻³¹ Kilogram

ElectronCharge

1.60218 × 10⁻¹⁹ Coulomb

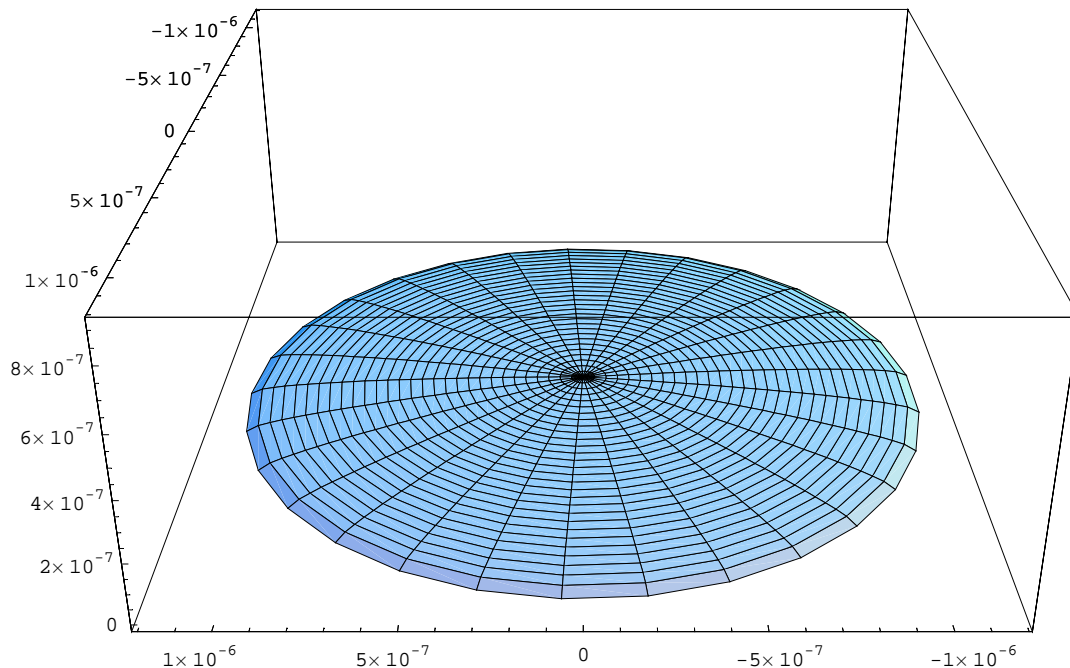
We animate the change in the charge distribution for the range of velocities 100 to 400 m/s:

```
Array[ChargeDist, {60}];
```

```

Do[
  Vel := 100 + 5 * i;
  rhoNot = ( 1.0545715964207855`^*-34 ) / ( 9.10938188`^*-31 * Vel );
  ChargeDens = FullSimplify[ ( 1.602176462`^*-19 / ( (2/3) * pi * (rhoNot)^3 ) ) * Sqrt[ (rhoNot)^2 - (rho)^2 ];
  ChargeDist[i] = CylindricalPlot3D[ChargeDens, {rho, 0, rhoNot}, {theta, 0, 2 pi},
    ViewPoint -> {0, 2, 1}, PlotRange -> {{-1.208604160266866`^*-6, 1.215390508631058`^*-6},
      {-1.2137774436356162`^*-6, 1.2137774436356162`^*-6},
      {-2.2831655476212466`^*-8, 9.36097874524711`^*-7}}, ImageSize -> 72 * 7],
  {i,
  1,
  60}]

```



Click **GIF** or **AVI** to see animation.

Next we will animate the free electron charge distribution using `ShadowPlot`, a different plotting function, the benefit being that it "shadows" the distribution into a plane. It requires cartesian coordinates,

```
CoordinatesFromCartesian[{x, y, z}, Cylindrical]
```

```
{Sqrt[x^2 + y^2], ArcTan[x, y], z}
```

Note that x and y must be less than ρ_0 , hence an infinitesimal constant is added to it in the `CartChargeDens` function. Note also that I am normalizing the scale of the graph to the 250 m/s electron radius.

$$\rho_{\text{notnorm}} = \left(\frac{1.0545715964207855 \cdot 10^{-34}}{9.10938188 \cdot 10^{-31} \cdot (250)} \right)$$

$$4.63071 \times 10^{-7}$$

Do[

Vel := 100 + 5 * i;

$$\rho_{\text{not}} = \left(\frac{1.0545715964207855 \cdot 10^{-34}}{9.10938188 \cdot 10^{-31} \cdot \text{Vel}} \right);$$

$$\text{CartChargeDens} = \text{FullSimplify} \left[\frac{1.602176462 \cdot 10^{-19}}{\frac{2}{3} \pi (\rho_{\text{not}})^3} \sqrt{(\rho_{\text{not}} + 1 \cdot 10^{-18})^2 - (x^2 + y^2)} \right];$$

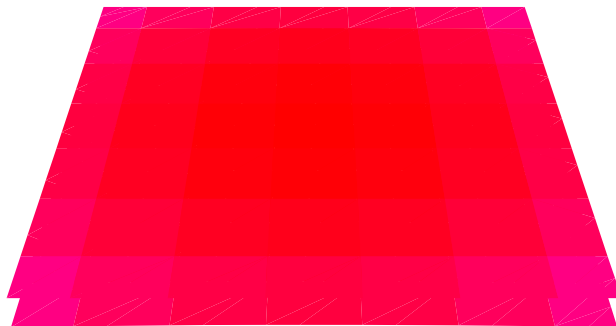
$$\text{CartChargeDist} = \text{ShadowPlot3D} \left[\text{CartChargeDens}, \left\{ x, -\frac{\rho_{\text{not}}}{\sqrt{2}}, \frac{\rho_{\text{not}}}{\sqrt{2}} \right\}, \right.$$

$$\left. \left\{ y, -\frac{\rho_{\text{not}}}{\sqrt{2}}, \frac{\rho_{\text{not}}}{\sqrt{2}} \right\}, \text{ShadowPosition} \rightarrow 1, \text{ViewPoint} \rightarrow \{0, 2, 1\}, \text{ShadowMesh} \rightarrow \text{False}, \right.$$

SurfaceMesh → False, Axes → False, Boxed → False, ImageSize → 72 * 5,

$$\text{PlotRange} \rightarrow \left\{ \left\{ -\frac{\rho_{\text{notnorm}}}{\sqrt{2}}, \frac{\rho_{\text{notnorm}}}{\sqrt{2}} \right\}, \left\{ -\frac{\rho_{\text{notnorm}}}{\sqrt{2}}, \frac{\rho_{\text{notnorm}}}{\sqrt{2}} \right\}, \{0, 6 \cdot 10^{-7}\} \right\},$$

{i,
1,
32}];

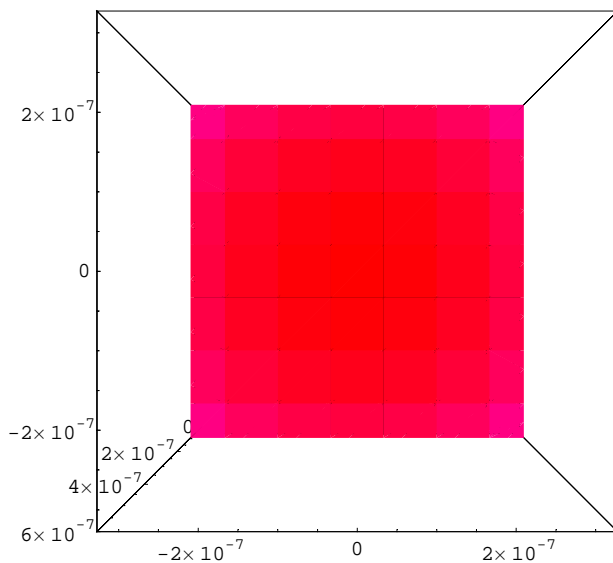


(Animation available in *Mathematica* only.) Viewed from the direction of propagation of the electron, see below

```

Do[
  Vel := 100 + 5 * i;
  rhoNot =  $\left( \frac{1.0545715964207855 \cdot 10^{-34}}{9.10938188 \cdot 10^{-31} \cdot \text{Vel}} \right)$ ;
  CartChargeDens = FullSimplify[ $\frac{1.602176462 \cdot 10^{-19}}{\frac{2}{3} \pi (\rho\text{not})^3} \sqrt{(\rho\text{not} + 1 \cdot 10^{-18})^2 - (x^2 + y^2)}$ ];
  CartChargeDist = ShadowPlot3D[CartChargeDens, {x, - $\frac{\rho\text{not}}{\sqrt{2}}$ ,  $\frac{\rho\text{not}}{\sqrt{2}}$ }, {y, - $\frac{\rho\text{not}}{\sqrt{2}}$ ,  $\frac{\rho\text{not}}{\sqrt{2}}$ },
    ShadowPosition → 1, ViewPoint → {0, 0, 2}, ShadowMesh → False, Axes → True,
    PlotRange → {{- $\frac{\rho\text{notnorm}}{\sqrt{2}}$ ,  $\frac{\rho\text{notnorm}}{\sqrt{2}}$ }, {- $\frac{\rho\text{notnorm}}{\sqrt{2}}$ ,  $\frac{\rho\text{notnorm}}{\sqrt{2}}$ }, {0,  $6 \cdot 10^{-7}$ }}],
  {i,
  1,
  32}];

```



Two static images from the above animations are:

```
Show[CartChargeDist, ViewPoint → {0, 2, 0}];
```

